THE THERMOELASTICITY PROBLEM FOR STRUCTURALLY NONUNIFORM SHELLS OF REGULAR STRUCTURE

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Structurally nonuniform shells exhibiting regular structure are encountered in various branches of engineering. Among these we include thin shells reinforced with a regular skeleton of closely positioned longitudinal or transverse reinforcement ribs (ribbed shells), a longitudinal-transverse system of stringers (waffle shells) and three-layered sandwich shells filled with a honeycomb structure (honeycomb shells), as well as various mesh and skeleton shells and plates. These and similar structurally nonuniform shells are used generally under conditions of intermittent heat fields, sharp drops of temperature, and the solution of problems of thermal conductivity and thermal elasticity for these structures is thus of practical interest.

The rapid oscillation in the coefficients of the equations and boundary conditions for such structural elements makes the problem in its exact formulation virtually unsolvable, even with a high-speed computer. This is associated with the need to develop certain approximate methods such as, for example, structural-anisotropy approaches. The regular structure of these shells makes it possible to resort to the asymptotic method of averaging the periodic structures in the design of such shells, and based on this method problems in the theory of elasticity, thermoconductivity, and thermal elasticity were dealt with in [1-3] for composition materials of regular structure as well as skeleton-structured materials. The averaging method [1-3] is also suitable for regularly nonuniform media exhibiting periodic structure in all three measurements. Structurally nonuniform shells, such as those considered in the present study, are neither one-dimensional nor two-dimensional composites (such as laminated or fiber materials), nor are they three-dimensional composites (such as granulated materials). Periodicity is exhibited only in the two tangential coordinates introduced at the middle surface of the shell or skin, with no such periodicity existing in the transverse coordinate. The limited dimensions of both period and shell thickness are commensurate in this case, and both force and heat boundary conditions are specified at the upper and lower surfaces of the shell. These features of the structural elements called for special (different from that discussed in [1-3]) asymptotic analysis of the corresponding three-dimensional problems for a thin layer, which would combine both the asymptotic transition from a three-dimensional problem to the two-dimensional problem of a shell and the transition (by the averaging method) from a nonuniform (composition) material to the equivalent uniform material (quasiuniform). The two-scale asymptotic method in this formulation for a plate was first proposed within the framework of the theory of elasticity in [4, 5]. The complete asymptotic expansions in the three-dimensional problem in the theory of elasticity for a thin plate with a thickness equal to the characteristic dimension of the nonuniformities were constructed and validated in [6]. In [7, 8] we find an asymptotic analysis of the three-dimensional problem from the theory of elasticity for a thin uniform plate with a rapidly oscillating thickness. A rather complete review of the papers devoted to the application of the averaging method in the problems of the mechanics of a deformed solid can be found in [9]. In [10, 11], within the scope of the theory of elasticity, without adopting any simplifying hypotheses, we find the asymptotic transition from the three-dimensional problem for a distorted regular nonuniform layer with a rapidly oscillating thickness to the model of an averaged shell. An analogous analysis has been undertaken in [12] for the problem of heat conduction in the case of the conditions of second- or third-kind heat exchange at the shell surfaces. A method has been developed in [13] which was applied to the quasistatic problem of thermal elasticity.

In the present paper we generalize the results derived in [10-13], and we present certain applications of the general model. In the first part of this study we cover the decisive

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relationships and the equations of the averaged shell. Its effective thermoelastic and thermophysical characteristics are determined from the solution of the auxiliary local problems on the periodicity cell. Based on the solution of the local problems and of the boundary-value problem for an averaged shell with great accuracy the three-dimensional local structure of the studied fields is reproduced. Applications are indicated for the general model with respect to various structural-nonuniform shells. Based on the solution of the local problems, explicit formulas are derived for all effective thermoelastic and thermophysical characteristics of drift, waffle, honeycomb, and mesh shells of regular structure.

1. Let us examine a nonuniform shell of regular structure exhibiting the periodicity cell Ω_ε which, in the orthogonal coordinate system α_1, α_2, γ, is given by the inequalities

$$-\varepsilon h_1/2 < \alpha_1 < \varepsilon h_1/2,\ -\varepsilon h_2/2 < \alpha_2 < \varepsilon h_2/2,\ \varepsilon z^- < \gamma < \varepsilon z^+,$$

where ε is the small parameter determining the thickness of the shell (skin); εh_1, εh_2 are the distances between the reinforcing elements; $z^+(y_1, y_2)$ and $z^-(y_1, y_2)$ are periodic functions of the variables $y_1 = \alpha_1/(\varepsilon h_1)$, $y_2 = \alpha_2/(\varepsilon h_2)$ specifying the shape of the reinforcements at the upper S^+ and lower S^- surfaces of the shell. We make the following notation: $z = \gamma/\varepsilon$, $y = (y_1, y_2)$, $\alpha = (\alpha_1, \alpha_2)$. We will assume that the elasticity coefficients $c_{ijmn}(y, z)$, as well as the coefficients of thermal conductivity $\lambda_{ij}(y, z)$, thermal expansion $\alpha_{ij}{}^\theta(y, z)$, and similar characteristics of the material are piecewise-smooth periodic functions for y_1, y_2 with the periodicity cell Ω: $\{y_1, y_2 \in (-1/2, 1/2),\ z \in (z^-, z^+)\}$, experiencing discontinuities of the first kind at a finite number of nonintersecting contact surfaces.

Following the method detailed in [10-13], we will present the components of the displacement vector and the temperature increment in the form of the asymptotic expansions

$$\begin{gathered} u_i = u_i^{(0)}(\alpha, t) + \varepsilon u_i^{(1)}(\alpha, t, y, z) + \varepsilon^2 u_i^{(2)}(\alpha, t, y, z) + \ldots, \\ \theta = \theta_1 + z\theta_2,\quad \theta_\nu = \theta_\nu^{(0)}(\alpha, t) + \varepsilon\theta_\nu^{(1)}(\alpha, t, y, z) + \ldots, \end{gathered} \tag{1.1}$$

where $u_i{}^{(\ell)}(\alpha, t, y, z)$, $\theta_\nu{}^{(\ell)}(\alpha, t, y, z)$ for $\ell = 1, 2, \ldots$, $\nu = 1, 2$ are the functions periodic for y_1, y_2 with the periodicity cell Ω.

It has been demonstrated in [10-13] that for the principal terms in (1.1) and in the corresponding expansions over ε for the components of the stress tensor and the heat-flex vector the relationships determining their local structure are valid:

$$u_1 = v_1(\alpha, t) - \varepsilon\frac{z}{A_1}\frac{\partial w(\alpha, t)}{\partial\alpha_1} + \varepsilon U_1^{\mu\nu}\omega_{\mu\nu} + \varepsilon^2 U_1^{*\mu\nu}\tau_{\mu\nu} + O(\varepsilon^3) \quad (1 \leftrightarrow 2), \tag{1.2}$$

$$u_3 = w(\alpha, t) + \varepsilon U_3^{\mu\nu}\omega_{\mu\nu} + \varepsilon^2 U_3^{*\mu\nu}\tau_{\mu\nu} + O(\varepsilon^3),$$

$$\theta = \theta_1^{(0)}(\alpha, t) + z\theta_2^{(0)}(\alpha, t) + \varepsilon\left(W_\mu\frac{1}{A_\mu}\frac{\partial\theta_1^{(0)}}{\partial\alpha_\mu} + W_\mu^*\frac{1}{A_\mu}\frac{\partial\theta_2^{(0)}}{\partial\alpha_\mu}\right) + O(\varepsilon^2);$$

$$\begin{gathered} \sigma_{ij} = b_{ij}^{\mu\nu}\omega_{\mu\nu} + \varepsilon b_{ij}^{*\mu\nu}\tau_{\mu\nu} - s_{ij}\theta_1^{(0)} - s_{ij}^*\theta_2^{(0)} + O(\varepsilon^2), \\ q_i = -\varepsilon^{-1}\lambda_{i3}\theta_2^{(0)} + q_i^{(0)} + O(\varepsilon),\quad q_i^{(0)} = -l_{i\nu}\frac{1}{A_\nu}\frac{\partial\theta_1^{(0)}}{\partial\alpha_\nu} - l_{i\nu}^*\frac{1}{A_\nu}\frac{\partial\theta_2^{(0)}}{\partial\alpha_\nu}. \end{gathered} \tag{1.3}$$

Here and below, summation is performed over identical indices, with the Latin indices taking on values of 1, 2, and 3, while the Greek indices take on values of 1 and 2; $A_1(\alpha)$, $A_2(\alpha)$ are the coefficients of the first quadratic form of the middle surface ($\gamma = 0$); $\omega_{11} = \varepsilon_1$, $\omega_{22} = \varepsilon_2$, $\omega_{12} = \omega_{21} = \omega/2$ represent the tensile and shearing strains; $\tau_{11} = \kappa_1$, $\tau_{22} = \kappa_2$, $\tau_{12} = \tau_{21} = \tau$ represent the flexural and torsional strains of the middle surface. These functions are expressed in terms of v_1, v_2, w by means of the familiar relationships from the theory of thin shells [11].

The following formulas are valid for the coefficients in relationships (1.3):

$$\begin{gathered} b_{ij}^{\mu\nu} = \frac{1}{h_\beta}c_{ijm\beta}\frac{\partial U_m^{\mu\nu}}{\partial\xi_\beta} + c_{ijm3}\frac{\partial U_m^{\mu\nu}}{\partial z} + c_{ij\mu\nu}, \\ b_{ij}^{*\mu\nu} = \frac{1}{h_\beta}c_{ijm\beta}\frac{\partial U_m^{*\mu\nu}}{\partial\xi_\beta} + c_{ijm3}\frac{\partial U_m^{*\mu\nu}}{\partial z} + zc_{ij\mu\nu}, \end{gathered} \tag{1.4}$$

$$s_{ij} = -\frac{1}{h_\beta} c_{ijm\beta} \frac{\partial V_m}{\partial \xi_\beta} - c_{ijm3} \frac{\partial V_m}{\partial z} + c_{ijmn} \alpha^\theta_{mn},$$

$$s^*_{ij} = -\frac{1}{h_\beta} c_{ijm\beta} \frac{\partial V^*_m}{\partial \xi_\beta} - c_{ijm3} \frac{\partial V^*_m}{\partial z} + z c_{ijmn} \alpha^\theta_{mn};$$

$$l_{i\mu} = \frac{1}{h_\beta} \lambda_{i\beta} \frac{\partial W_\mu}{\partial \xi_\beta} + \lambda_{i3} \frac{\partial W_\mu}{\partial z} + \lambda_{i\mu}, \tag{1.5}$$

$$l^*_{i\mu} = \frac{1}{h_\beta} \lambda_{i\beta} \frac{\partial W^*_\mu}{\partial \xi_\beta} + \lambda_{i3} \frac{\partial W^*_\mu}{\partial z} + z\lambda_{i\mu}.$$

The functions $U_m^{\mu\nu}$, $U_m^{*\mu\nu}$, V_m, V_m^*, W_μ, and W_μ^*, contained in relationships (1.2)-(1.5), depend on $\xi_1 = A_1 y_1$, $\xi_2 = A_2 y_2$, and z. With respect to ξ_1, ξ_2, they are periodic solutions (with the periods A_1, A_2, respectively) of the following local problems on the periodicity cell:

$$\frac{1}{h_\beta} \frac{\partial}{\partial \xi_\beta} b^{\mu\nu}_{i\beta} + \frac{\partial}{\partial z} b^{\mu\nu}_{i3} = 0, \tag{1.6}$$

$$\left(\frac{1}{h_\beta} N^\pm_\beta b^{\mu\nu}_{i\beta} + N^\pm_3 b^{\mu\nu}_{i3}\right)\Big|_{z=z\pm} = 0 \quad (b^{\mu\nu}_{ij} \leftrightarrow b^{*\mu\nu}_{ij} \leftrightarrow s_{ij} \leftrightarrow s^*_{ij});$$

$$\frac{1}{h_\beta} \frac{\partial l_{\beta\mu}}{\partial \xi_\beta} + \frac{\partial l_{3\mu}}{\partial z} = 0, \tag{1.7}$$

$$\left(\frac{1}{h_\beta} N^\pm_\beta l_{\beta\mu} + N^\pm_3 l_{3\mu}\right)\Big|_{z=z\pm} = 0 \quad (l_{i\mu} \leftrightarrow l^*_{i\mu})$$

[$N_i^\pm$ is the component normal to the surfaces $z = z^\pm(y)$].

At the material characteristic discontinuity surfaces we find fulfillment of the continuity conditions which correspond to ideal contact (n_i is the component normal to the discontinuity surface):

$$[U^{\mu\nu}_m] = 0, \quad \left[\frac{n_\beta}{h_\beta} b^{\mu\nu}_{i\beta} + n_3 b^{\mu\nu}_{i3}\right] = 0 \tag{1.8}$$

$$(U^{\mu\nu}_m \leftrightarrow U^{*\mu\nu}_m \leftrightarrow V_m \leftrightarrow V^*_m,\ b^{\mu\nu}_{ij} \leftrightarrow b^{*\mu\nu}_{ij} \leftrightarrow s_{ij} \leftrightarrow s^*_{ij});$$

$$[W_\mu] = 0, \quad \left[\frac{n_\beta}{h_\beta} l_{\beta\mu} + n_3 l_{3\mu}\right] = 0 \quad (W_\mu \leftrightarrow W^*_\mu,\ l_{i\mu} \leftrightarrow l^*_{i\mu}). \tag{1.9}$$

Local problems (1.4), (1.6), (1.8) and (1.5), (1.7), (1.9) have single solutions accurate to the constant terms. This nonuniqueness is removed by imposition of the conditions

$$\langle U^{\mu\nu}_m \rangle_\xi = 0 \quad \text{or} \quad z = 0$$
$$(U^{\mu\nu}_m \leftrightarrow U^{*\mu\nu}_m \leftrightarrow V_m \leftrightarrow V^*_m \leftrightarrow W_\mu \leftrightarrow W^*_\mu). \tag{1.10}$$

The subscript ξ indicates integration over the coordinates ξ_1, ξ_2.

Averaging relationships (1.3) by means of integration over the volume Ω, we obtain (r = 0.1)

$$\langle z^r \sigma_{ij} \rangle = \langle z^r b^{\mu\nu}_{ij} \rangle \omega_{\mu\nu} + \varepsilon \langle z^r b^{*\mu\nu}_{ij} \rangle \tau_{\mu\nu} - \tag{1.11}$$
$$- \langle z^r s_{ij} \rangle \theta^{(0)}_1 - \langle z^r s^*_{ij} \rangle \theta^{(0)}_2 + O(\varepsilon^2);$$

$$\langle z^r q^{(0)}_i \rangle = -\langle z^r l_{i\nu} \rangle \frac{1}{A_\nu} \frac{\partial \theta^{(0)}_1}{\partial \alpha_\nu} - \langle z^r l^*_{i\nu} \rangle \frac{1}{A_\nu} \frac{\partial \theta^{(0)}_2}{\partial \alpha_\nu}. \tag{1.12}$$

Relationships (1.11) and (1.12) represent the equations of state for the averaged shell, while the coefficients of these relationships represent its effective thermoelastic and thermophysical characteristics. In this case, on the basis of (1.4)-(1.10), it has been demonstrated that

$$\langle b^{\mu\nu}_{3i} \rangle = 0, \ \langle z b^{\mu\nu}_{3i} \rangle = 0 \quad (b^{\mu\nu}_{3i} \leftrightarrow b^{*\mu\nu}_{3i} \leftrightarrow s_{3i} \leftrightarrow s^*_{3i} \leftrightarrow l_{3\mu} \leftrightarrow l^*_{3\mu}), \tag{1.13}$$
$$\langle b^{\mu\nu}_{\beta\delta} \rangle = \langle b^{\beta\delta}_{\mu\nu} \rangle, \quad \langle z b^{\mu\nu}_{\beta\delta} \rangle = \langle b^{*\beta\delta}_{\mu\nu} \rangle,$$

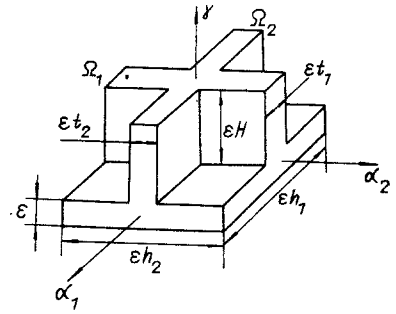

Fig. 1

$$\langle zb_{\beta\delta}^{*\mu\nu}\rangle = \langle zb_{\mu\nu}^{*\beta\delta}\rangle, \quad \langle l_{\mu\nu}\rangle = \langle l_{\nu\mu}\rangle,$$
$$\langle zl_{\mu\nu}\rangle = \langle l_{\nu\mu}^{*}\rangle, \quad \langle zl_{\mu\nu}^{*}\rangle = \langle zl_{\nu\mu}^{*}\rangle.$$

Relationships (1.13) provide for the symmetry of the matrices comprised of the coefficients of the equations of state for the averaged shell.

Systems of resolving equations have been derived in [10-13] for the functions v_1, v_2, w and $\theta_1{}^{(0)}$, $\theta_2{}^{(0)}$. Let us note that, as demonstrated in [11-13], in the case of a uniform material and with constant shell thickness ($z^{\pm} = \pm 1/2$) the averaged model reduces to the relationships taken from the theory of the thermoelasticity of anisotropic shells, with the following formulas valid for the linear forces, the moments, and the integral temperature characteristics:

$$N_1 = \varepsilon\langle\sigma_{11}\rangle,\ M_1 = \varepsilon^2\langle z\sigma_{11}\rangle\ (1 \leftrightarrow 2),$$
$$N_{12} = \varepsilon\langle\sigma_{12}\rangle, \quad M_{12} = \varepsilon^2\langle z\sigma_{12}\rangle, \quad T = \theta_1^{(0)}, \quad T^* = \theta_2^{(0)}/2.$$

Let us dwell in some detail on certain applications of the general model to the design of structural-nonuniform shells of regular structure, these having been fabricated out of a uniform isotropic material.

2. Let us examine the waffle shell with a periodicity cell consisting of three mutually perpendicular elements (Fig. 1). The approximate analytical solution of local problems (1.4)-(1.10) for a cell of the indicated form can be found in the assumption that the thickness of each of the cell elements is small in comparison to the other dimensions, i.e., under the conditions $t_1 \ll h_1$, $t_2 \ll h_2$, h_1, $h_2 \sim H$.

The approximation method for the problems in the theory of thermoelasticity which involves problems from the theory of plates and shells, such as that used in the solution of local problems (1.4)-(1.10), was proposed and verified in [14, 15] in determining the effective characteristics of the small-cell skeletal constructions of periodic structure. In combination with the above-described general model of an averaged shell, this method makes it possible to obtain in explicit form and with adequate accuracy the effective characteristics for a large number of reinforced shells, such as those used in actual practice. In this case, for all effective characteristics different from zero and contained in the equations of state (1.11) and (1.12), with consideration of relationships (1.13), we will obtain

$$\langle b_{11}^{11}\rangle = \frac{E}{1-\nu^2} + EF_2, \quad \langle b_{22}^{22}\rangle = \frac{E}{1-\nu^2} + EF_1, \tag{2.1}$$
$$\langle b_{22}^{11}\rangle = \frac{E\nu}{1-\nu^2}, \quad \langle b_{12}^{12}\rangle = \frac{E}{2(1+\nu)}, \quad \langle b_{11}^{*11}\rangle = ES_2, \quad \langle b_{22}^{*22}\rangle = ES_1,$$
$$\langle zb_{11}^{*11}\rangle = \frac{E}{12(1-\nu^2)} + EJ_2, \quad \langle zb_{22}^{*22}\rangle = \frac{E}{12(1-\nu^2)} + EJ_1, \quad \langle zb_{22}^{*11}\rangle = \frac{E\nu}{12(1-\nu^2)},$$
$$\langle zb_{12}^{*12}\rangle = \frac{E}{24(1+\nu)}\left[1 + H^3\left(\frac{t_1}{h_1} + \frac{t_2}{h_2}\right) - K_1 - K_2\right],$$

$$\langle s_{11}\rangle = \frac{E\alpha^\theta}{1-\nu} + E\alpha^\theta F_2, \quad \langle s_{22}\rangle = \frac{E\alpha^\theta}{1-\nu} + E\alpha^\theta F_1,$$

$$\langle s_{11}^*\rangle = E\alpha^\theta S_2, \quad \langle s_{22}^*\rangle = E\alpha^\theta S_1,$$

$$\langle zs_{11}^*\rangle = \frac{E\alpha^\theta}{12(1-\nu)} + E\alpha^\theta J_2, \quad \langle zs_{22}^*\rangle = \frac{E\alpha^\theta}{12(1-\nu)} + E\alpha^\theta J_1,$$

$$\langle l_{11}\rangle = \lambda + \lambda F_2, \quad \langle l_{22}\rangle = \lambda + \lambda F_1, \quad \langle l_{11}^*\rangle = \lambda S_2, \quad \langle l_{22}^*\rangle = \lambda S_1,$$

$$\langle zl_{11}^*\rangle = \frac{\lambda}{12}\left(1 + \frac{t_1 H^3}{h_1} + 12J_2 - K_1\right),$$

$$\langle zl_{22}^*\rangle = \frac{\lambda}{12}\left(1 + \frac{t_2 H^3}{h_2} + 12J_1 - K_2\right).$$

Here $K_1 = \frac{96H^4}{\pi^5 A_1 h_1}\sum_{n=1}^{\infty}\frac{[1-(-1)^n]}{n^5}\,\mathrm{th}\,\frac{\pi n A_1 t_1}{2H}$ $(1 \leftrightarrow 2)$; E, ν, λ, α^θ are the characteristics of the material;

F_1, F_2 are the cross-sectional areas; S_1, S_2 are the static moments; J_1, J_2 are the inertial moments of the cross sections of the reinforcement elements Ω_1, Ω_2 relative to the middle surface of the skin ($z = 0$), calculated in the coordinate system y_1, y_2, z. For these the following formulas are valid (see Fig. 1):

$$F_1 = \frac{t_1 H}{h_1}, \quad S_1 = \frac{t_1(H^2+H)}{2h_1}, \quad J_1 = \frac{t_1(4H^3+6H^2+3H)}{12h_1} \quad (1 \leftrightarrow 2). \tag{2.2}$$

Formulas (2.1) and (2.2) for the effective rigidity moduli (the elastic portion of the characteristics) are in good agreement with the familiar relationships from the structural-anisotropic theory of reinforced plates. The formula for torsional rigidity $\langle zb_{12}{}^{*12}\rangle$ is beyond the scope of this theory, according to which the following is assumed:

$$\langle zb_{12}^{*12}\rangle = \frac{E}{24(1+\nu)}\left(1 + H\frac{t_1^3}{h_1} + H\frac{t_2^3}{h_2}\right). \tag{2.3}$$

The correction factors for formulas (2.1) in comparison with (2.3) are significant in the case of high rigidity ribs. For example, when $A_1 = A_2 = 1$, $\nu = 0.3$, $H = 20$, $h_1 = h_2 = 60$, $t_1 = t_2 = 2$ from (2.1) we have $\langle zb_{12}{}^{*12}\rangle/E = 0.1922$ (a correction factor of −5.3%), and with $H = 10$, $h_1 = h_2 = 10$, $t_1 = t_2 = 1$ we have $\langle zb_{12}{}^{*12}\rangle/E = 0.0921$ (a correction of −4.3%).

To check on the errors in formulas (2.1), we undertook a more exact (numerical) solution of local problems (1.4)-(1.10), and this showed that the error in formulas (2.1) and (2.2) (with the exception of the formula for $\langle zb_{22}{}^{*11}\rangle$) amounts to less than 1%, and these can therefore be used in practice with an accuracy suitable for the majority of engineering calculations. The greatest correction factors were obtained in this case for the effective rigidity moduli $\langle b_{22}{}^{*11}\rangle$ and $\langle zb_{22}{}^{*11}\rangle$, for which when $\nu = 0.3$ it follows from (2.1) that $\langle b_{22}{}^{*11}\rangle = 0$, $\langle zb_{22}{}^{*11}\rangle/E = 0.0275$. For purposes of comparison, the results from the numerical calculation of these moduli in seven variants (with $\nu = 0.3$) can be found in Table 1.

Formulas (2.1) and (2.2) can be used to determine the effective characteristics of ribbed shells. For example, in the case of stiffening ribs directed along the $O\alpha_1$ coordinate line, in Fig. 1 we should remove the reinforcing element Ω_1, while in formulas (2.1) we should assume that $t_1 = 0$ and correspondingly that $F_1 = S_1 = J_1 = K_1 = 0$.

3. Let us examine a three-layered shell consisting of upper and lower stress-bearing layers and a honeycomb filler of a four-sided structure (Fig. 2). On the basis of the approximate analytic solution of local problems (1.4)-(1.10), obtained in analogy with the previous case, for all effective characteristics of the honeycomb shell different from zero [with consideration of relationships (1.13)] we have

$$\langle b_{11}^{11}\rangle = \frac{2E_0}{1-\nu_0^2} + EF_2, \quad \langle b_{22}^{22}\rangle = \frac{2E_0}{1-\nu_0^2} + EF_1, \tag{3.1}$$

$$\langle b_{22}^{11}\rangle = \frac{2E_0\nu_0}{1-\nu_0^2}, \quad \langle b_{12}^{12}\rangle = \frac{E_0}{1+\nu_0}, \quad \langle zb_{11}^{*11}\rangle = \frac{2E_0 J_3}{1-\nu_0^2} + E\frac{t_2 H^3}{12h_2},$$

$$\langle zb_{22}^{*22}\rangle = \frac{2E_0 J_3}{1-\nu_0^2} + E\frac{t_1 H^3}{12h_1}, \quad \langle zb_{22}^{*11}\rangle = \frac{2E_0\nu_0}{1-\nu_0^2}J_3,$$

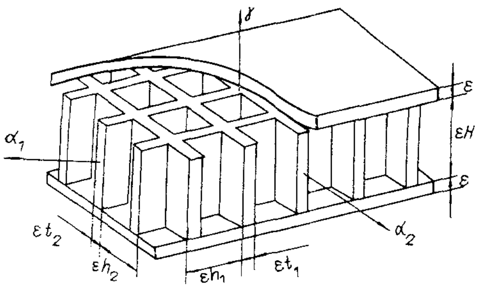

Fig. 2

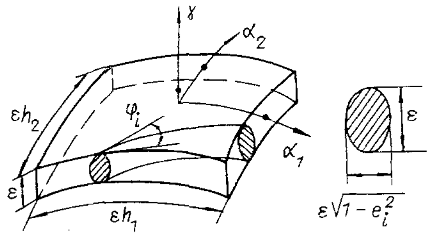

Fig. 3

$$\langle zb_{12}^{*12}\rangle = \frac{E_0 J_3}{1+\nu_0} + \frac{E}{24(1+\nu)}\left(\frac{H^3 t_1}{h_1} + \frac{H^3 t_2}{h_2} - K_1 - K_2\right),$$

$$\langle s_{11}\rangle = \frac{2E_0\alpha_0^\theta}{1-\nu_0} + E\alpha^\theta F_2, \quad \langle s_{22}\rangle = \frac{2E_0\alpha_0^\theta}{1-\nu_0} + E\alpha^\theta F_1,$$

$$\langle zs_{11}^{*}\rangle = \frac{2E_0\alpha_0^\theta}{1-\nu_0} J_3 + E\alpha^\theta \frac{t_2 H^3}{12h_2},$$

$$\langle zs_{22}^{*}\rangle = \frac{2E_0\alpha_0^\theta}{1-\nu_0} J_3 + E\alpha^\theta \frac{t_1 H^3}{12h_1},$$

$$\langle l_{11}\rangle = 2\lambda_0 + \lambda F_2, \quad \langle l_{22}\rangle = 2\lambda_0 + \lambda F_1,$$

$$\langle zl_{11}^{*}\rangle = 2\lambda_0 J_3 + \frac{\lambda}{12}\left(\frac{t_1 H^3}{h_1} + \frac{t_2 H^3}{h_2} - K_1\right),$$

$$\langle zl_{22}^{*}\rangle = 2\lambda_0 J_3 + \frac{\lambda}{12}\left(\frac{t_1 H^3}{h_1} + \frac{t_2 H^3}{h_2} - K_2\right),$$

where E_0, ν_0, λ_0, α_0^θ represent the characteristics of the material in the upper and lower stress-carrying layers; E, ν, λ, α^θ are the characteristics of the material making up the foil of the honeycomb filler; F_1, F_2, K_1, K_2 have been determined above; $J_3 = (3H^2 + 6H + 4)/12$. The first terms in formulas (3.1) represent the contribution of the stress-carrying layers, while the second terms represent the contribution made by the honeycomb filler.

TABLE 1

Variant No.	Periodicity cell parameters	$\langle b_{22}^{*11}\rangle/E$	$\langle zb_{22}^{*11}\rangle/E$
1	$H = 10,\ h_1 = h_2 = 10$ $t_1 = t_2 = 1$	–0,0648	–0,3705
2	$H = 10,\ h_1 = h_2 = 10$ $t_1 = 1,\ t_2 = 0{,}5$	–0,0432	–0,2378
3	$H = 10,\ h_1 = h_2 = 20$ $t_1 = t_2 = 0{,}5$	–0,0078	–0,0207
4	$H = 8,\ h_1 = h_2 = 30$ $t_1 = t_2 = 0{,}8$	–0,0028	0,0132
5	$H = 8,\ h_1 = h_2 = 30$ $t_1 = 0{,}8,\ t_2 = 0{,}4$	–0,0019	0,0180
6	$H = 20,\ h_1 = h_2 = 20$ $t_1 = t_2 = 0{,}5$	–0,0648	–0,7406
7	$H = 20,\ h_1 = h_2 = 60$ $t_1 = t_2 = 2$	–0,0277	–0,3030

4. Let us take a look at the mesh shell of regular structure, formed by N families of elliptical sections parallel to each other, and these, in particular, of circular lateral cross section. Here ε denotes the thickness of the shell and φ_i represents the angle formed by the sections of the i-th family with the coordinate line $O\alpha_1$, γ_i is the volumetric content of the sections in the i-th family in the periodicity cell, e_i is the eccentricity of the transverse cross section of the sections in the i-th family, and E_i, ν_i, λ_i, α_i^θ are the characteristics of the material in the sections of the i-th family. Figure 3 shows the periodicity cell and one of the sections in the i-th family.

On the basis of an analytical solution of local problems (1.4)-(1.10) for the sections forming the mesh, and based on the principle of separation for the averaged operator [1], we obtained the following formulas for all of the (different from zero) effective characteristics of the mesh shell:

$$\langle b_{\beta\delta}^{\mu\varkappa}\rangle = \sum_{i=1}^{N} E_i b_i \gamma_i, \quad \langle z b_{\beta\delta}^{*\mu\varkappa}\rangle = \sum_{i=1}^{N} E_i b_i \left(1 + \frac{c_i}{1+\nu_i}\right)\frac{\gamma_i}{16}, \tag{4.1}$$

$$\langle s_{\beta\delta}\rangle = \sum_{i=1}^{N} E_i \alpha_i^\theta s_i \gamma_i, \quad \langle z s_{\beta\delta}^{*}\rangle = \langle s_{\beta\delta}\rangle/16,$$

$$\langle l_{\beta\delta}\rangle = \sum_{i=1}^{N} \lambda_i s_i \gamma_i, \quad \langle z l_{\beta\delta}^{*}\rangle = \sum_{i=1}^{N} \lambda_i l_i \gamma_i/16.$$

The parameters b_i, c_i, s_i, and ℓ_i, in (4.1), depend on the set of indices $\beta\delta\mu\kappa$ and are determined from the following formulas:

for $\beta\delta\mu\varkappa = 1111$

$$b_i = A_1^4 B_i^{-4}\cos^4\varphi_i, \quad c_i = 2A_2^4 \operatorname{tg}^2\varphi_i\,(1-e_i^2)\,\Delta_i,$$

$$s_i = \sqrt{b_i}, \quad l_i = [A_1^2\cos^2\varphi_i + A_1^2A_2^2(1-e_i^2)]\Delta_i \quad (\beta\delta = 11);$$

for $\beta\delta\mu\varkappa = 2222$

$$b_i = A_2^4 B_i^{-4}\sin^4\varphi_i, \quad c_i = 2A_1^4 \operatorname{ctg}^2\varphi_i\,(1-e_i^2)\,\Delta_i,$$

$$s_i = \sqrt{b_i}, \quad l_i = [A_2^2\sin^2\varphi_i + A_1^2A_2^2(1-e_i^2)]\Delta_i \quad (\beta\delta = 22);$$

for $\beta\delta\mu\varkappa = 1212$

$$b_i = A_1^2A_2^2 B_i^{-4}\cos^2\varphi_i\sin^2\varphi_i,$$

$$c_i = \frac{1}{2}\left(A_1^4\operatorname{ctg}^2\varphi_i + A_2^4\operatorname{tg}^2\varphi_i - 2A_1^2A_2^2\right)(1-e_i^2)\,\Delta_i,$$

$$s_i = \sqrt{b_i}, \quad l_i = A_1A_2\Delta_i\cos\varphi_i\sin\varphi_i \quad (\beta\delta = 12, 21);$$

for $\beta\delta\mu\varkappa = 1122,\ 2211$

$$b_i = A_1^2A_2^2 B_i^{-4}\cos^2\varphi_i\sin^2\varphi_i, \quad c_i = -2A_1^2A_2^2(1-e_i^2)\,\Delta_i;$$

for $\beta\delta\mu\varkappa = 1112,\ 1211$

$$b_i = A_1^3A_2 B_i^{-4}\cos^3\varphi_i\sin\varphi_i, \quad c_i = A_2^2\left(A_2^2\operatorname{tg}^2\varphi_i - A_1^2\right)(1-e_i^2)\,\Delta_i;$$

for $\beta\delta\mu\varkappa = 1222,\ 2212$

$$b_i = A_1A_2^3 B_i^{-4}\cos\varphi_i\sin^3\varphi_i, \quad c_i = A_1^2\left(A_1^2\operatorname{ctg}^2\varphi_i - A_2^2\right)(1-e_i^2)\,\Delta_i.$$

Here

$$B_i^2 = A_1^2\cos^2\varphi_i + A_2^2\sin^2\varphi_i; \quad \Delta_i = [B_i^2 + A_1^2A_2^2(1-e_i^2)]^{-1}.$$

The formulas for the effective characteristics $\langle b_{\beta\delta}^{\mu\kappa}\rangle$, $\langle s_{\beta\delta}\rangle$, and $\langle \ell_{\beta\delta}\rangle$ are valid for an arbitrary transverse cross section. The obliquely symmetric portion of the effective characteristics is equal to zero because of the symmetry of the elliptic sectional cross section under consideration here, relative to the middle surface of the cell.

Let us note that in the particular cases of rectangular, rhombic, and triangular mesh shells, the formulas which follow out of (4.1), within the framework of the theory of elasticity, were derived in [16], while within the framework of the thermal conductivity problem these formulas were derived in [17].

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LITERATURE CITED

1. N. S. Bakhvalov and G. P. Panasenko, Averaging Processes in Periodic Media. Mathematical Problems in the Mechanics of Composition Materials [in Russian], Nauka, Moscow (1984).
2. E. Sanchez-Palencia, "Nonhomogeneous media and vibration theory," Lect. Notes Phys., No. 127 (1980).
3. B. E. Podbedrya, The Mechanics of Composition Materials [in Russian], Izd-vo MGU, Moscow (1984).
4. D. Caillerie, "Plaques élastiques minces à structure périodique de période et d'épaisseur comparables," C. R. Acad. Sci., Ser. II, 294, No. 3 (1982).
5. D. Caillerie, "Thin elastic and periodic plates," Math. Meth. in Appl. Sci., 6, No. 2 (1984).
6. G. P. Panasenko and M. V. Reztsov, "Averaging the three-dimensional problem in the theory of elasticity for a nonuniform plate," Dokl. Akad. Nauk SSSR, 294, No. 5 (1987).
7. R. V. Kohn and M. Vogelius, "A new model for thin plates with rapidly varying thickness," Int. J. Solids Struct., 20, No. 4 (1984).
8. R. V. Kohn and M. Vogelius, "A new model for thin plates with rapidly varying thickness. II. A convergence proof," Q. Appl. Math., 43, No. 1 (1985).
9. A. L. Kalamkarov, B. A. Kudryavtsev, and V. Z. Parton, "The asymptotic method of averaging in the mechanics of regularly structured composite materials," in: Results in Science and Engineering, Series: The Mechanics of a Deformed Solid, Vol. 19 [in Russian], VINITI, Moscow (1987).
10. V. Z. Parton, A. L. Kalamkarov, and B. A. Kudryavtsev, "The stressed-strained state of a twisted anisotropically nonuniform layer of periodic structure with wavelike surfaces," Dokl. Akad. Nauk SSSR, 291, No. 6 (1986).
11. A. L. Kalamkarov, B. A. Kudryavtsev, and B. Z. Parton, "The problem of a twisted layer of composite material with wavy surfaces of periodic structure," PMM, 51, No. 1 (1987).
12. A. L. Kalamkarov, "The thermal conductivity of a twisted nonuniform anisotropic layer of periodic structure with wavy surfaces," JEP, 52, No. 5 (1987).
13. A. L. Kalamkarov, B. A. Kudryavtsev, and B. Z. Parton, "The thermal elasticity of a regular nonuniform twisted layer with wavy surfaces," PMM, 51, No. 6 (1987).
14. A. G. Kolpakov, "The determination of averaged characteristics for elastic skeletons," PMM, 49, No. 6 (1985).
15. A. G. Kolpakov, "Averaged characteristics of thermoelastic skeletons," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 6 (1987).
16. A. L. Kalamkarov, "Determining the effective characteristics of mesh shells and plates of periodic structure," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 2 (1987).
17. A. L. Kalamkarov, "The thermal conductivity of mesh shells and plates of regular structure," JEP, 54, No. 3 (1988).